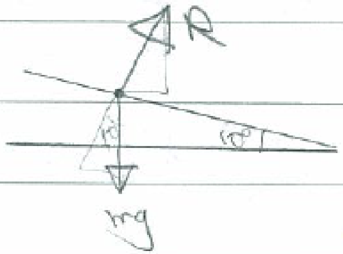
M3 - June 2003

1. $ME_A = ME_B + WD$

$$\frac{\lambda x^2}{2a} = 0 + \frac{2}{3}mg\,AB$$

$$\frac{4mga^2}{2a \cdot 4} = \frac{2}{3}mg\,AB$$

$$AB = \frac{3a}{4}$$

2. 

$\downarrow$ ~~$R = mg\cos\theta$~~ $\quad R\cos\theta = mg$

$$R = \frac{mg}{\cos\theta}$$

$\rightarrow$ [$F = ma$]

$$R\sin\theta = \frac{m \cdot 18^2}{r}$$

$$r = \frac{324}{g\tan 10} = 190\text{m (2 s.f.)}$$

3\. a) $[F = ma]$

$$\frac{1}{10}x(4-3x) = 0.2\, v\frac{dv}{dx}$$

$$\int_6^x 4x - 3x^2\, dx = 2\int_0^v v\, dv$$

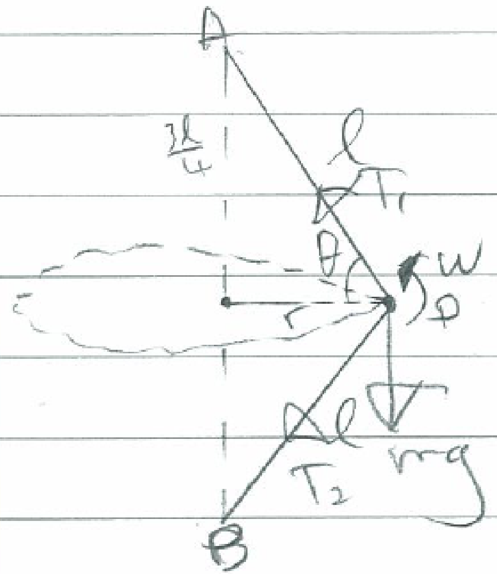
$$\left[2x^2 - x^3\right]_6^x = \left[v^2\right]_0^v$$

$$v^2 = 2x^2 - x^3 + 144$$

b) When $x = 0$, $v^2 = 144$

$v = 12\text{ms}^{-1}$

4.



a) ↑ $mg - T_1 \sin\theta + T_2 \sin\theta = 0$

$T_2 \times \frac{3l}{4l} = T_1 \cdot \frac{3l}{4l} - mg$

$T_2 = T_1 - \frac{4mg}{3}$ ①

← [F = ma]

$T_1 \cos\theta + T_2 \cos\theta = mrw^2$

$T_1 \cos\theta = m l \cos\theta \, w^2 - T_2 \cos\theta$

$T_1 = ml\omega^2 - T_1 + \frac{4mg}{3}$

$2T_1 = \frac{1}{3}m(3l\omega^2 + 4g)$

$T_1 = \frac{1}{6}m(3l\omega^2 + 4g)$

b) $T_2 = T_1 - \frac{4mg}{3}$

$= \frac{1}{2}ml\omega^2 + \frac{2mg}{3} - \frac{4}{3}mg$

$= \frac{1}{2}ml\omega^2 - \frac{2}{3}mg$

$= \frac{1}{6}m(3l\omega^2 - 4g)$

c) $T_2 \geq 0$

$\frac{1}{6}m(3l\omega^2 - 4g) \geq 0$

$3l\omega^2 \geq 4g$

$\omega^2 \geq \frac{4g}{3l}$

5 a) 

$T = \frac{\lambda x}{d} = \frac{12x}{0.6} = 20x$

$\rightarrow$ [$F = ma$]

$-T = 0.8\ddot{x}$

$-20x = 0.8\ddot{x}$

$\ddot{x} = -25x$

$\omega^2 = 25$

$\omega = 5$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$ s

b) $a_{max} = \omega^2 r = 25 \times 0.25 = 6.25$ m s^{-2}

c) $x = 0.25\cos 5t$

$v = -1.25\sin 5t$

At $t=3$ $v = -1.25\sin 10 = 0.68$ m s^{-1}

d) Away from O

6 – a) $ME_A = ME_C$

$$\frac{1}{2}mu^2 + mg(a - a\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2ga - 2ga\cos\theta$$

$\swarrow$ $\boxed{F = ma}$

$$mg\cos\theta - R = \frac{mv^2}{r}$$

$$mg\cos\theta = \frac{m}{a}\left(u^2 + 2ga - 2ga\cos\theta\right)$$

$$g\cos\theta = \frac{u^2}{a} + 2g - 2g\cos\theta$$

$$3g\cos\theta = \frac{u^2}{a} + 2g$$

$$\cos\theta = \frac{u^2}{3ga} + \frac{2}{3}$$

b) $MÉ_A = ME_P$

$\frac{1}{2}mu^2 + m2ag = \frac{1}{2}m\frac{9ag}{2}$

$u^2 = \frac{9ag}{2} - 4ag = \frac{ag}{2}$

$\cos\theta = \frac{ag}{2\times 3ga} + \frac{2}{3} = \frac{1}{6} + \frac{2}{3} = \frac{5}{6}$

$\theta = 34°$ (2sf)

7- a) $V = \pi\int_0^2 y^2\,dx = \frac{\pi}{4}\int_0^2 (x-2)^4\,dx = \frac{\pi}{4}\left[\frac{1}{5}(x-2)^5\right]_0^2$

$= \frac{\pi}{20}(0+32) = \frac{8\pi}{5}$ cm^3

b) $\pi \int_0^2 y^2 x \, dx = \frac{8\pi}{5} \bar{x}$

$\frac{\pi}{4} \int_0^2 x(x-2)^4 \, dx = \frac{8\pi}{5} \bar{x}$

Let $u = x$ $\quad \frac{dv}{dx} = (x-2)^4$

$\frac{du}{dx} = 1$ $\quad v = \frac{1}{5}(x-2)^5$

$\frac{32\bar{x}}{5} = \left[\frac{x}{5}(x-2)^5\right]_0^2 - \frac{1}{5}\int_0^2 (x-2)^5 \, dx$

$= -\frac{1}{5\times 6}\left[(x-2)^6\right]_0^2$

$= -\frac{1}{30}(-64) = 3$

$\frac{32\bar{x}}{5} = \frac{32}{15}$

$\bar{x} = \frac{5 \times 32}{\ldots} = \frac{1}{3}$ cm

$\overline{15+82} \quad 5$

c) B) $\frac{1}{3} \times$ ~~[scribble]~~ $\times 2W + 8F - 4 \times 10W = 0$

$\frac{2}{3}$ ~~[illegible]~~ $W + 8F - 40W = 0$

$8F = \frac{118}{3}W$

$F = \frac{59}{12}W$